Several Improvements in The Methods for Estimating Directional Spectra Observed with A Submerged Doppler-Type Directional Wave Meter

Noriaki Hashimoto

Kyushu University, Japan

Irregular sea waves and Directional spectrum

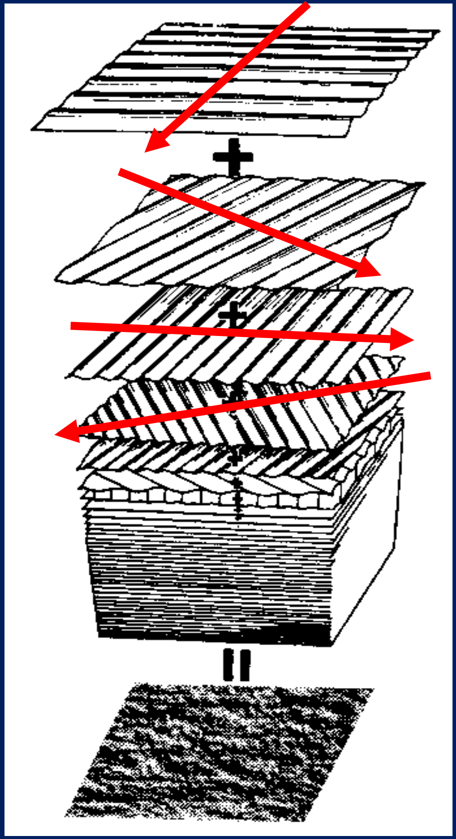
Mathematical expression

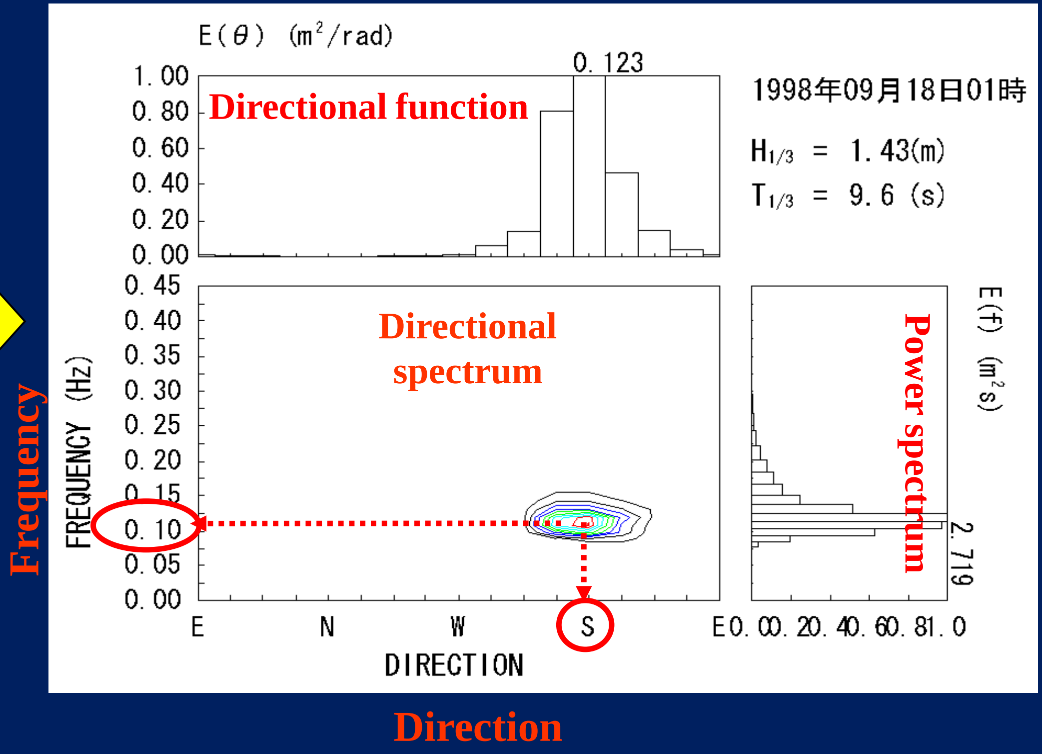
$$\eta(\mathbf{x},t)=\int_{\mathbf{k}}\int_{\omega} dZ(\mathbf{k},\omega)\exp\{i(\mathbf{kx}-\omega t)\}$$

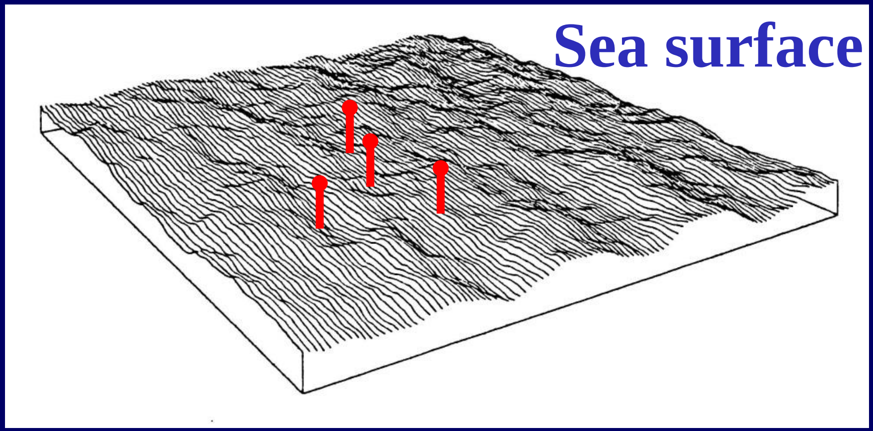
Directional spectrum is an important parameter to express the irregular wave characteristics

Definition of directional spectrum

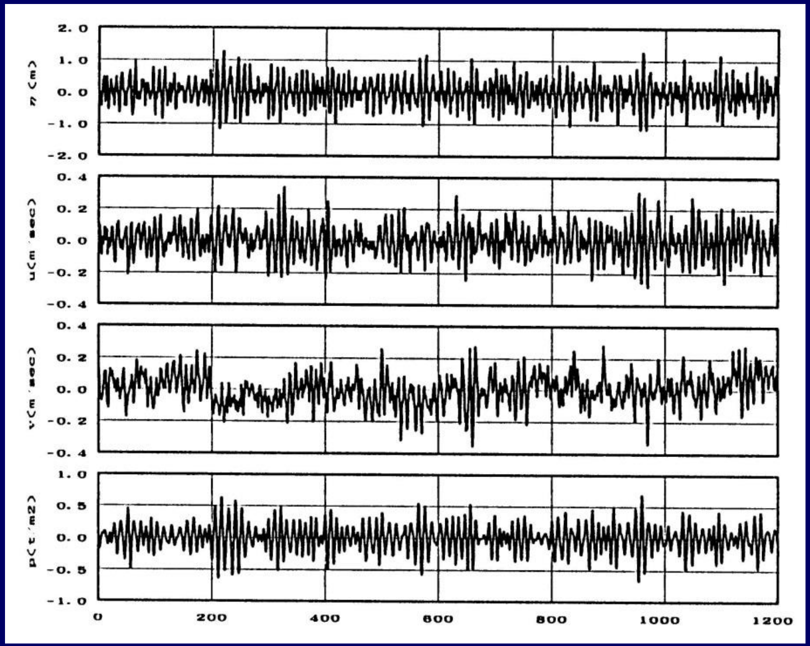
$$S(\mathbf{k},\omega)d\mathbf{k}d\omega = < dZ^*(\mathbf{k},\omega)dZ(\mathbf{k},\omega) >$$







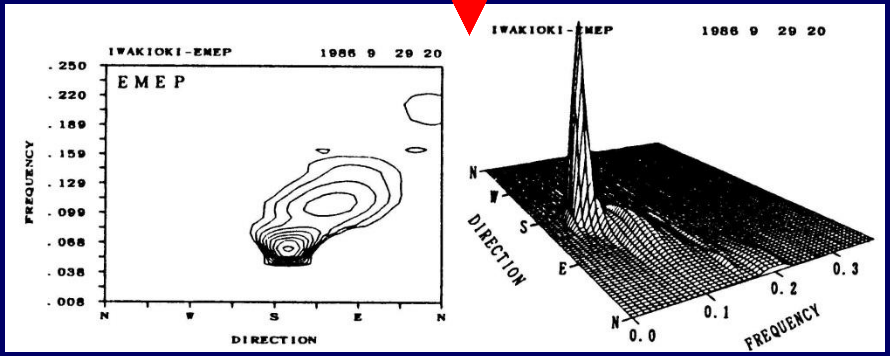
【wave observation】



Time series of wave data

（at least 3 components）

【directional spectrum analysis】



Directional spectrum

☆Fundamental equation for directional spectrum estimation

$$\boxed{\boxed{\Phi_{mn}(f)} = \int_0^{2\pi} H_m(f,\theta) H_n^*(f,\theta) \boxed{S(f,\theta)}\, d\theta}$$

where $\boxed{\Phi_{mn}(\theta)}$: cross-power spectrum

$\boxed{S(f,\theta)} \geq 0$: directional spectrum

$H_m(f,\theta)$: transfer function

When we measure N-quantities, $\xi_n(t)$ (n=1,･･･,N), related to water surface elevation, $\eta(t)$, we get a set of cross-power spectra, $\Phi_{i,j}(f)$ (i, j=1,…,M) ;

$$\begin{pmatrix} \Phi_{1,1}(f), & \Phi_{1,2}(f), & \cdots & \Phi_{1,N}(f) \\ \Phi_{2,1}(f), & \Phi_{2,2}(f), & \cdots & \Phi_{2,N}(f) \\ \vdots & \vdots & \ddots & \vdots \\ \Phi_{N,1}(f), & \Phi_{N,2}(f), & \cdots & \Phi_{N,N}(f) \end{pmatrix}$$

where, $\Phi_{i,j}(f)$ and $\Phi_{j,i}(f)$ are complex conjugate each other.

When several wave quantities ξ_i are measured, and the cross-spectra $\Phi_{1,1}, \ \Phi_{1,2},\ldots,\Phi_{N,N}$ between each wave quantity are estimated, the following **simultaneous integral equations** are given to each cross-spectrum.

$$
\left.
\begin{aligned}
\Phi_{1,1}(f) &= \int_0^{2\pi} H_1(f,\theta)\,H_1(f,\theta)\,\boxed{S(f,\theta)}\,d\theta \\
\Phi_{1,2}(f) &= \int_0^{2\pi} H_1(f,\theta)H_2(f,\theta)\,\boxed{S(f,\theta)}\,d\theta \\
&\ \vdots \\
\Phi_{N,N}(f) &= \int_0^{2\pi} H_N(f,\theta)\,H_N(f,\theta)\,\boxed{S(f,\theta)}\,d\theta
\end{aligned}
\right\}
$$

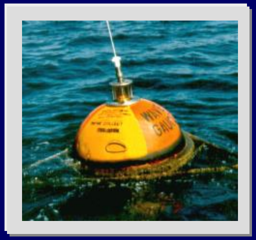
unknown

The directional spectrum ***S*** **(** $\boldsymbol{f, \theta}$ **)** can be estimated as a non-negative solution for the above simultaneous equations.

In essentials, the number of unknown parameters of directional seas is much larger than that of equations !!

Incomplete inverse problem !!

For the estimation of directional wave spectrum, several methods have been proposed.



- Direct Fourier Transformation Method (DFTM)
- Parametric Method
- Maximum Likelihood Method (MLM)
- Extended Maximum Likelihood Method (EMLM)
- Maximum Entropy Method (MEM)

- **Maximum Entropy Principle Method (MEP), 1985**
- **Bayesian Directional Spectrum Estimation Method (BDM), 1987**
- **Extended Maximum Entropy Principle Method (EMEP), 1993**

Direct Fourier Transformation Method (DFTM)

$$\boxed{\Phi_{m,n}(\omega;\mathbf{r}) = \int_{\mathbf{k}} S(\mathbf{k},\omega)\exp(i\mathbf{k}\mathbf{r})d\mathbf{k}} \rightleftarrows S(\mathbf{k},\omega) = \frac{1}{(2\pi)^2}\int_{\mathbf{r}} \Phi_{m,n}(\omega;\mathbf{r})\exp(-i\mathbf{k}\mathbf{r})d\mathbf{r}$$

$$\Downarrow$$

$$S(\mathbf{k},\omega) = \alpha \sum_m \sum_n \Phi_{m,n}(\omega)\exp\left\{ i\mathbf{k}(\mathbf{x}_n - \mathbf{x}_m)\right\}$$

Although the method's computation is easy, the directional resolution is low and a negative energy distribution sometimes occurs.

Parametric method

$$G(\theta \mid f) = a_0(f) + \sum_n \left\{ a_n(f)\cos n\theta + b_n(f)\sin n\theta \right\}$$

$$G(\theta \mid f) = \alpha(f)\cos^{2S(f)}\left\{\frac{\theta - \theta(f)}{2}\right\}$$

$$G(\theta \mid f) = \frac{\exp\left[a(f)\cos\left\{\theta - \theta(f)\right\}\right]}{2\pi I_0\left\{a(f)\right\}}$$

The estimated directional spectrum is easily obtained by substituting any of these formulations into the fundamental equations and determining its parameters.

However, the disadvantage of this method lies in the fact that the real directional spectrum cannot be estimated unless the applied formulation is indeed a suitable representation.

Maximum Likelihood Method (MLM) (Capon, 1969)

Extended Maximum Likelihood Method (EMLM) (Isobe, at.al., 1984)

$$\Phi_{mn}(\omega) = \int_{\mathbf{k}} \boxed{H_m(\mathbf{k},\omega) H_n{}^*(\mathbf{k},\omega)} S(\mathbf{k},\omega)\, d\mathbf{k}$$

$$\hat{S}(\boldsymbol{k},\omega) = \sum_m \sum_n \alpha_{mn}(\boldsymbol{k}) \Phi_{mn}(\omega)$$

$$S(\mathbf{k},\omega) = \frac{1}{(2\pi)^2} \int_{\mathbf{r}} \Phi_{m,n}(\omega;\mathbf{r}) \exp(-i\mathbf{k}\mathbf{r})\, d\mathbf{r}$$

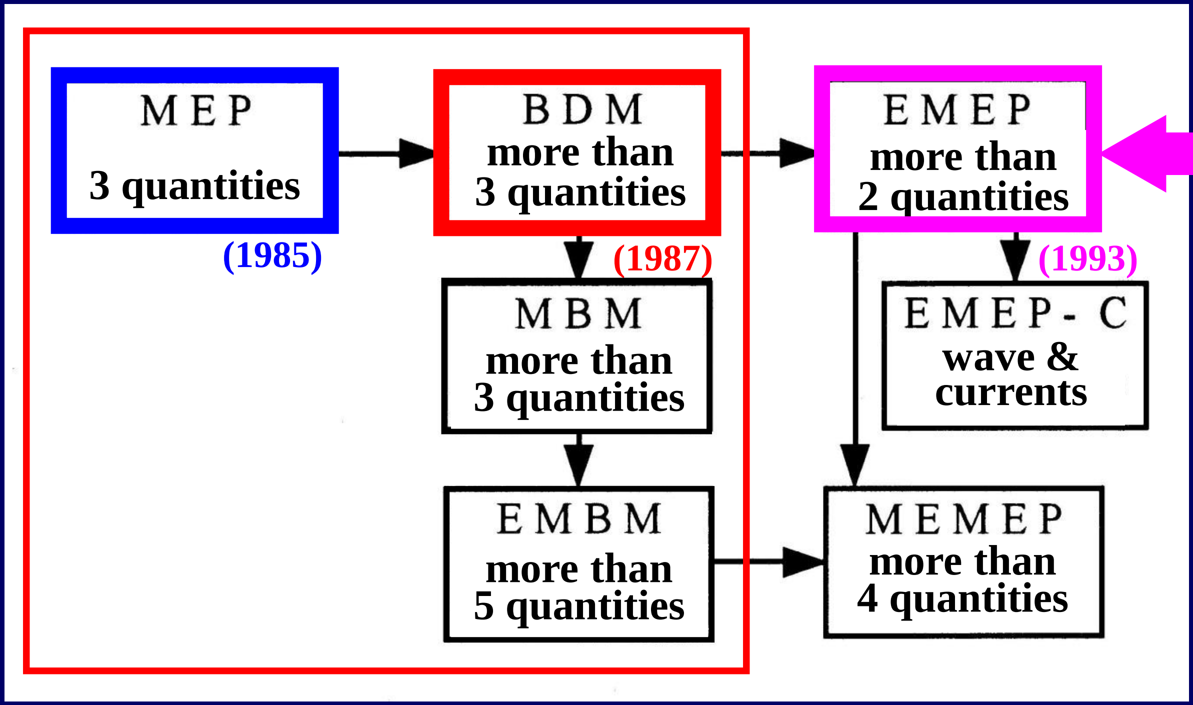
$$\hat{S}(\mathbf{k},\omega) = \int_{\mathbf{k}'} S(\mathbf{k}',\omega) \boxed{w(\mathbf{k},\mathbf{k}')} d\mathbf{k}'$$

where, $\boxed{w(\mathbf{k},\mathbf{k}')} = \sum_m \sum_n \alpha_{mn}(\mathbf{k}) H_m{}^*(\mathbf{k}',\omega) H_n(\mathbf{k}',\omega)$

If $w(\mathbf{k},\mathbf{k}') \to \delta(\mathbf{k},\mathbf{k}')$, then $\hat{S}(\mathbf{k},\omega) \to S(\mathbf{k},\omega)$!!

$$\boxed{\hat{S}(\mathbf{k},\omega) = \kappa \Big/ \left[\sum_m \sum_n \Phi_{mn}{}^{-1}(\omega) \boxed{H_m{}^*(\mathbf{k},\omega) H_n(\mathbf{k},\omega)} \right]}$$

Development of directional spectrum estimation methods in Japan



Standard directional spectrum estimation method in Japan

These measurements furnished the minimum data necessary for estimating the directional spectrum in the full directional range of $[0,2\pi]$. → **MEP**

Accuracy of directional spectrum estimation is much improved when more than 3 wave quantities are measured → **BDM**

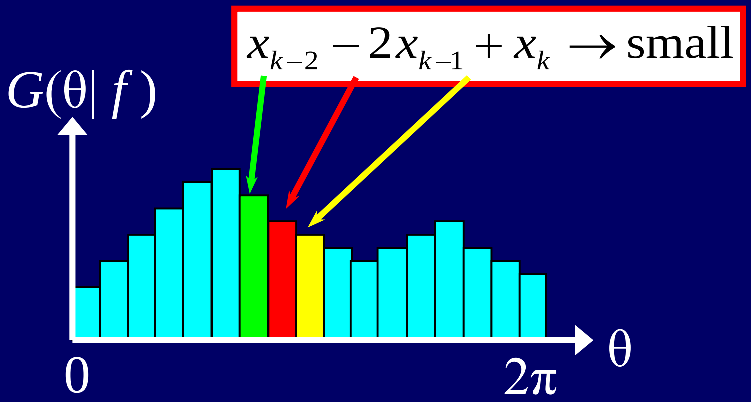
MEP: Maximum Entropy Method (1985)
BDM: Bayesian Directional spectrum estimation method (1987)
MBM: BDM applied to incident and reflected wave field
(reflection coefficient. is known) (1987)
EMBM: BDM applied to incident and reflected wave field
(reflection coefficient is unknown) (1987)
EMEP: Extended MEP (1993)
MEMEP: EMEP applied to incident and reflected wave field
(reflection coefficient is unknown) (1993)
EMEP-C: EMEP applied to wave & currents field (1994)

Bayesian method for directional spectrum estimation (BDM, Hashimoto, et.al, 1987)

piece-wise constant function

$$G(\theta \mid f) \approx \sum_{k=1}^{K} \exp\{x_k(f)\} I_k(\theta)$$

$$\text{where, } I_k(\theta) = \begin{cases} 1 : (k-1)\Delta\theta \le \theta < k\Delta\theta \\ 0 : \quad \text{otherwise} \end{cases}$$



unknown parameters x_k are estimated by minimizing the following equation

$$\sum_{j=1}^{J} \left\{ \Phi_j - \sum_{k=1}^{K} \alpha_{j,k} \exp(x_k) \right\}^2 + \boxed{u^2} \left\{ \sum_{k=1}^{K} (x_k - 2x_{k-1} + x_{k-2})^2 \right\} \rightarrow \min.$$

hyper-parameter $\boxed{u^2}$ is selected by minimizing the following ABIC

$$\mathrm{ABIC} = -2 \ln \int L(z_1, \cdots, z_K; \sigma^2) p(z_1, \cdots, z_K \mid u^2, \sigma^2) d\mathbf{x} \rightarrow \min.$$

Search method of optimal parameter u

$$u = ab^{m-1} \quad (m = 1, 2, \cdots)$$

$$a = 1.0, \; b = 0.5 \Rightarrow b = 0.9$$

Function form of MEP (Hashimoto, et.al., 1985)

$$G(\theta \mid f) = \exp\{a_0(f) + a_1(f)\cos\theta + b_1(f)\sin\theta + a_2(f)\cos 2\theta + b_2(f)\sin 2\theta\}$$

Function form of BDM (Hashimoto, et.al., 1987)

$$G(\theta \mid f) \approx \sum_{k=1}^{K} \exp\{x_k(f)\} I_k(\theta)$$

Common point;

Exponential function (non-negative)

Extended MEP (EMEP) (Hashimoto, et.al., 1993)

$$G(\theta \mid f) = \exp\left[a_0(f) + \sum_{n=1}^{N}\{a_n(f)\cos n\theta + b_n(f)\sin n\theta\}\right]$$

The power of the exp-function is expressed by the Fourier series with arbitrary order ***N***.

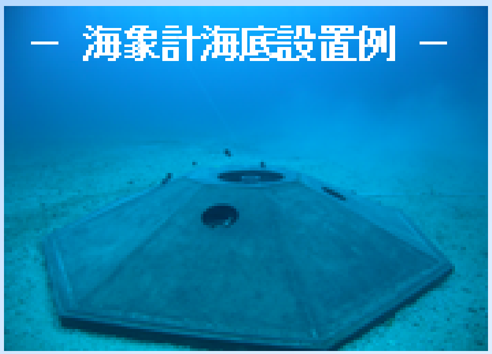
The order *N* is determined by minimizing AIC (Akaike's Information Criterion).

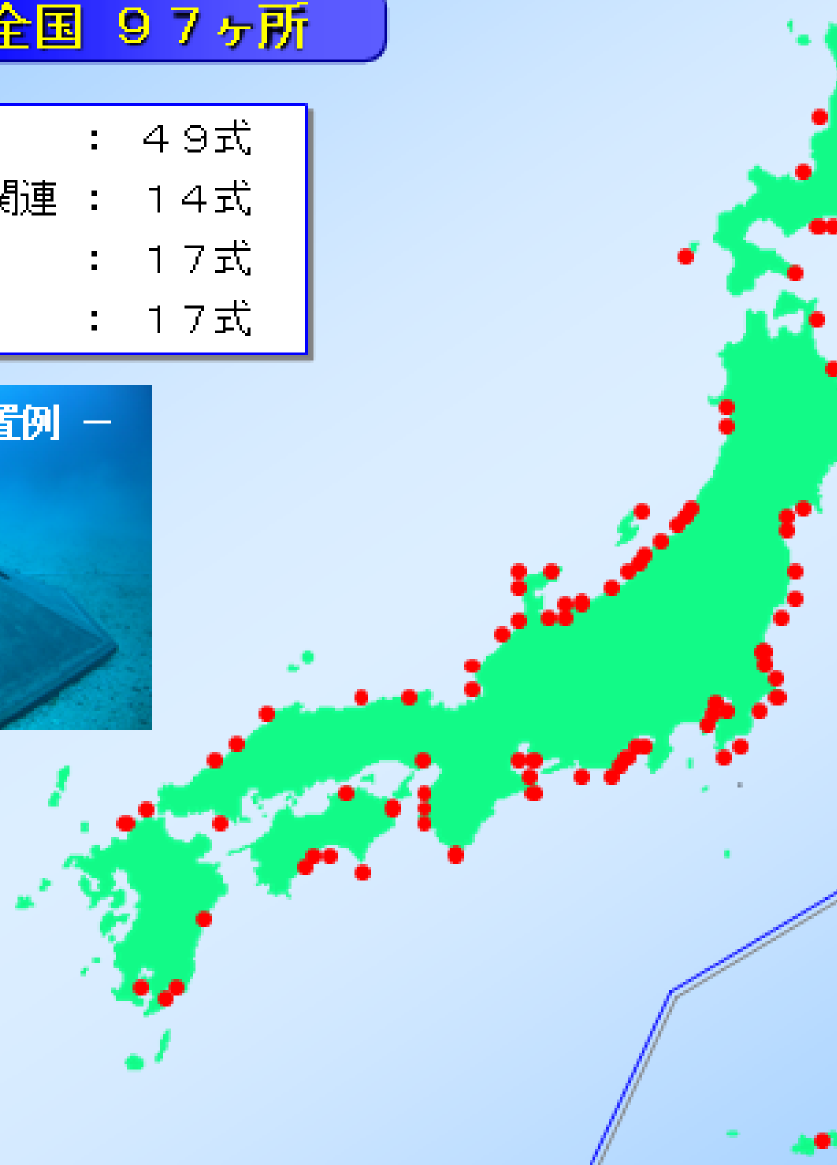
$$\mathrm{AIC} = M(\ln 2\pi\hat{\sigma}^2 + 1) + 2(2N+1) \rightarrow \mathrm{Min.}$$

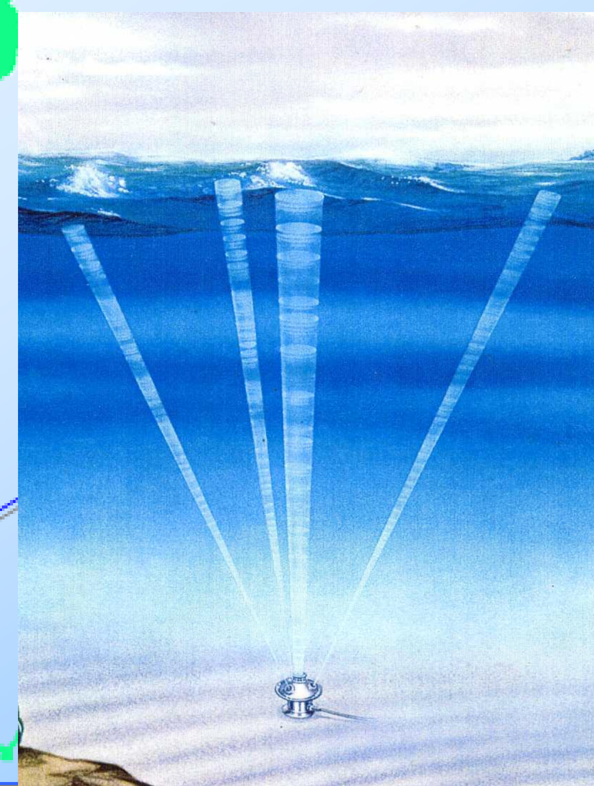
Submerged Ultrasonic Doppler-type Directional Wave Meter (1992)

納入実績数　：　全国　９７ヶ所

国土交通省港関係	：	４９式
開発局･総合事務局関連	：	１４式
地方自治体	：	１７式
電力・石油・ガス	：	１７式







２０１２．３月現在

$$\Phi_{mn}(f) = \int_0^{2\pi} H_m(f,\theta) H_n{}^*(f,\theta) S(f,\theta)\, d\theta$$

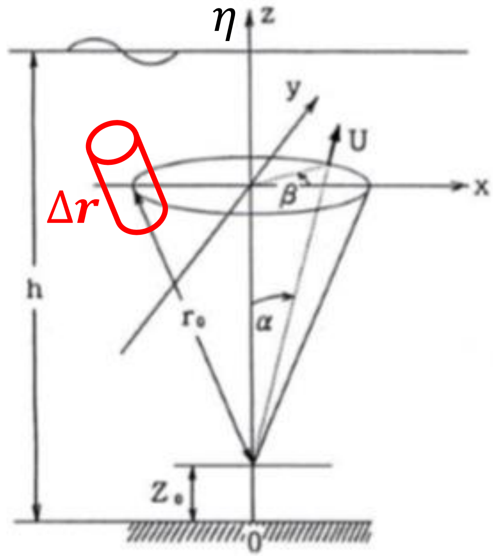
In the polar coordinates, the water particle velocity U in the direction r with the coordinate (α, β, γ) is given by

$$\Phi_{mn}(f) = \int_0^{2\pi} H_m(f,\theta) H_n{}^*(f,\theta) S(f,\theta)\, d\theta$$

where η is the water surface elevation and H the transfer function between U and η.
The function H can be expressed using linear wave theory as

$$\begin{aligned} H_0(\alpha,\beta,r,d,z_0;\omega,\theta) = \frac{\omega \exp(-i\omega \boxed{\Delta t})}{\sinh kd} \big[&\cosh\{k(r\cos\alpha + z_0)\} \\ &\times \sin\alpha \cos(\theta-\beta) - i \sinh\{k(r\cos\alpha + z_0)\} \\ &\times \cos\alpha \big] \exp\{ikr \sin\alpha \cos(\theta-\beta)\}, \end{aligned}$$

where **Δt is the time lag between the measurement of each velocity component and that of η.**



The current velocity component detected by the Doppler-type wave meter is not a water particle velocity at a specified location, but instead **an average velocity in a volume of known width Δr.**

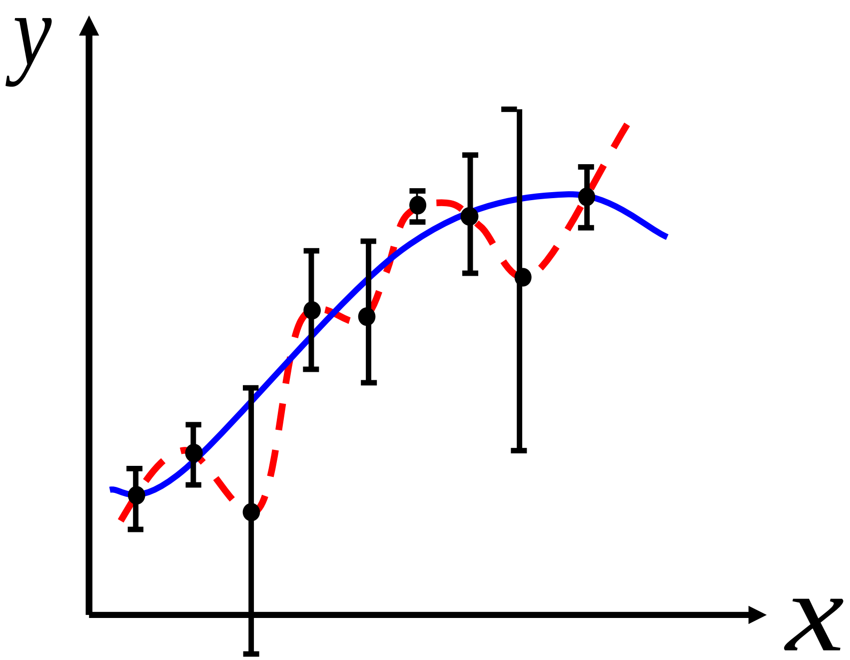
$$\overline{H}(\alpha,\beta,r_0,\Delta r,d,z_0;\omega,\theta) = \frac{1}{\Delta r}\int_{r_0-\Delta r/2}^{r_0+\Delta r/2} H_0(\alpha,\beta,r,d,z_0;\omega,\theta)dr$$

$$= \boxed{\frac{-i\omega \exp(-i\omega\Delta t)}{\Delta r k \sinh kd}\Big[\cosh\{k(r\cos\alpha + z_0)\}\exp\{ikr\sin\alpha\cos(\theta-\beta)\}\Big]_{r_0-\Delta r/2}^{r_0+\Delta r/2}},$$

Regression Model: $\quad y_i = F(x_i) + \varepsilon_i \quad ; \quad (i = 1, \cdots\cdots, N)$

Weighted Least Squares Method :

$$E = \sum_{i=1}^{N} W_i^{\,2} \varepsilon_i^{\,2} = \sum_{i=1}^{N} W_i^{\,2} \left\{ y_i - F(x_i) \right\}^2 \longrightarrow \min$$



(1) $W_i \equiv 1$

(2) $W_i \equiv 1 / y_i$

(3) $W_i \equiv 1 / \sigma_i$

Weights for weighted least squares method

Normalization of the cross-spectra

$$W_{mn} = \left[\Phi_{mm}(f)\Phi_{nn}(f)\right]^{1/2}$$ ← Geometric mean of the spectra

(Hashimoto,et.al, 1987)

The standard deviations of error of the co- and quadrature-spectra

$$W_{mn} = \sigma[\hat{C}_{mn}(f)] \approx \left[\left\{\Phi_{mm}(f)\Phi_{nn}(f) + C_{mn}(f)^2 - Q_{mn}(f)^2\right\}/2N_a\right]^{1/2}$$

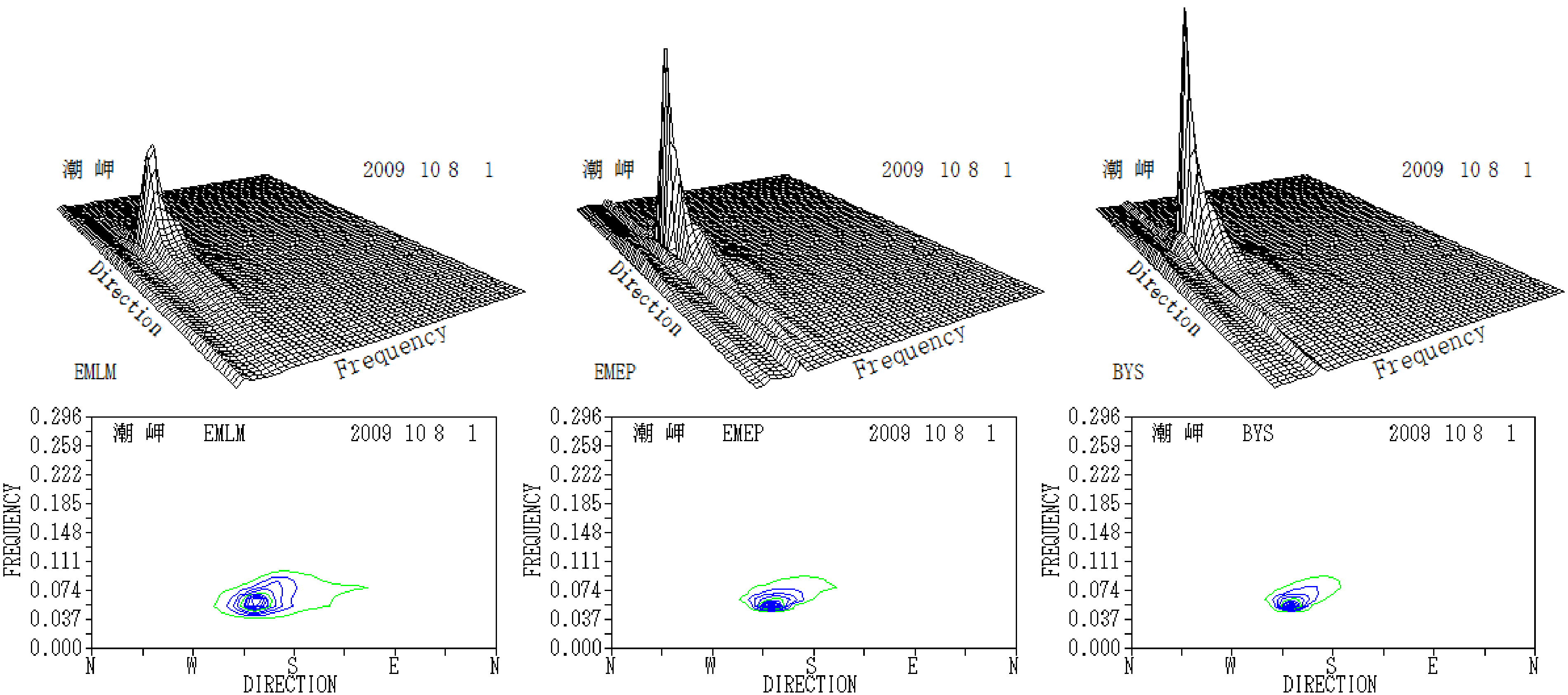
$$W_{mn} = \sigma[\hat{Q}_{mn}(f)] \approx \left[\left\{\Phi_{mm}(f)\Phi_{nn}(f) - C_{mn}(f)^2 + Q_{mn}(f)^2\right\}/2N_a\right]^{1/2}$$

where, $\Phi_{mn}(f) = C_{mn}(f) - iQ_{mn}(f)$

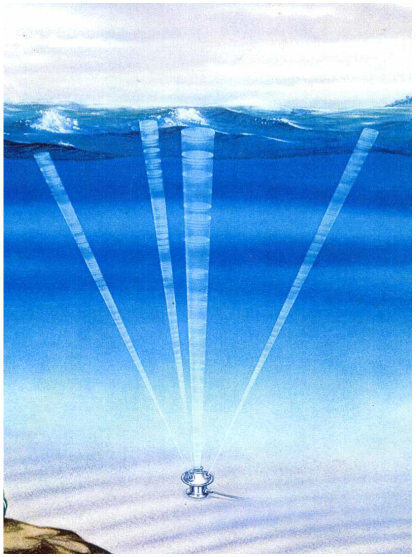
(Hashimoto,et.al, 1993)

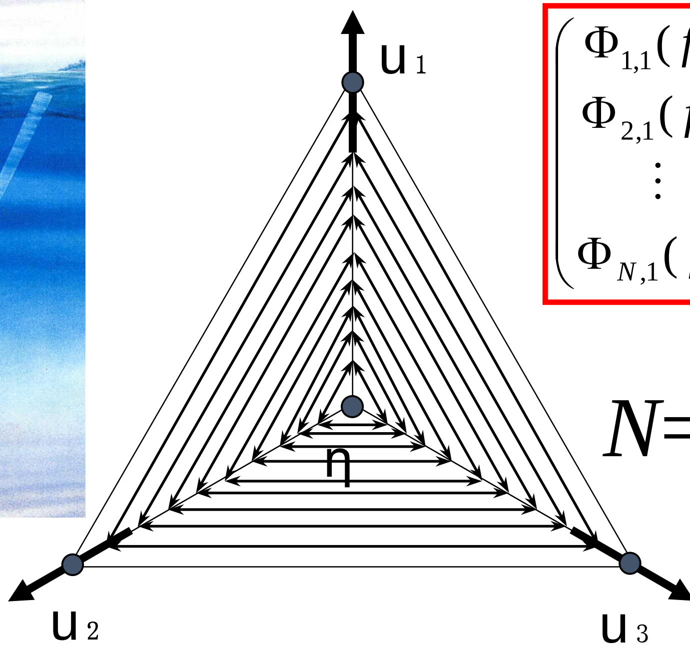
Directional spectra at Shiono-misaki observed with Doppler-type directional wave meter (3 layer measurement)

(Long traveled Swell from Typhoon)



EMLM EMEP Bayes





N=10

$$\begin{pmatrix} \Phi_{1,1}(f), & \Phi_{1,2}(f), & \cdots & \Phi_{1,N}(f) \\ \Phi_{2,1}(f), & \Phi_{2,2}(f), & \cdots & \Phi_{2,N}(f) \\ \vdots & \vdots & \ddots & \vdots \\ \Phi_{N,1}(f), & \Phi_{N,2}(f), & \cdots & \Phi_{N,N}(f) \end{pmatrix}$$

$$\left.\begin{aligned} \Phi_{1,1}(f) &= \int_0^{2\pi} H_{1,1}(f,\theta)\, S(f,\theta)\, d\theta \\ \Phi_{1,2}(f) &= \int_0^{2\pi} H_{1,2}(f,\theta)\, S(f,\theta)\, d\theta \\ &\vdots \\ \Phi_{N,N}(f) &= \int_0^{2\pi} H_{N,N}(f,\theta)\, S(f,\theta)\, d\theta \end{aligned}\right\}$$

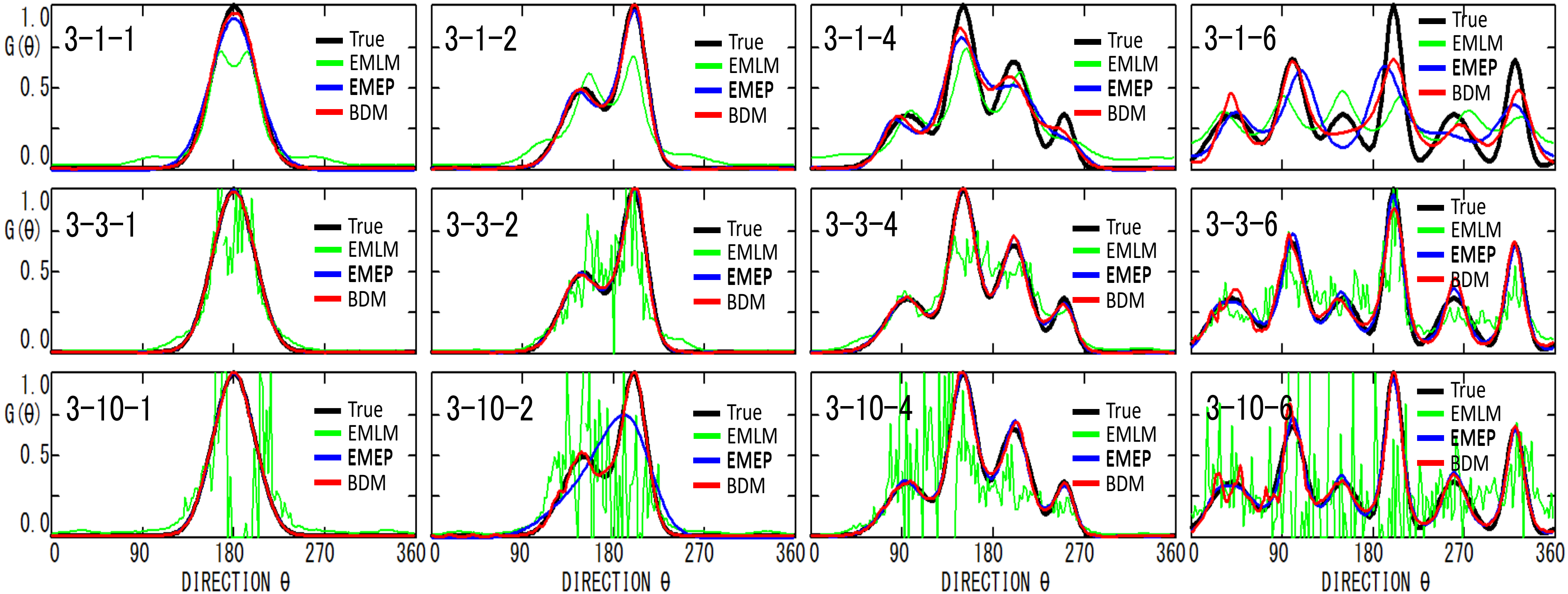
Fundamental equations can be set from each component of $N \times N$ matrices composed of cross power spectra.

$N \times (N+1)/2$ equations can be set when complex conjugate components are excluded from the matrices.

1 layer measurement; 4 components quantities → 10 equations

3 layer measurement; 10 component quantities → 55 equations

10 layer measurement; 31 component quantities → **496 equations !**



Examples of direction functions using 1 layer data (upper panels), 3 layers data (middle panels), and 10 layers data (lower panels) of DWM.

(A direction function having 1 peak, 2 peaks, 4 peaks, and 6 peaks in order from the left)

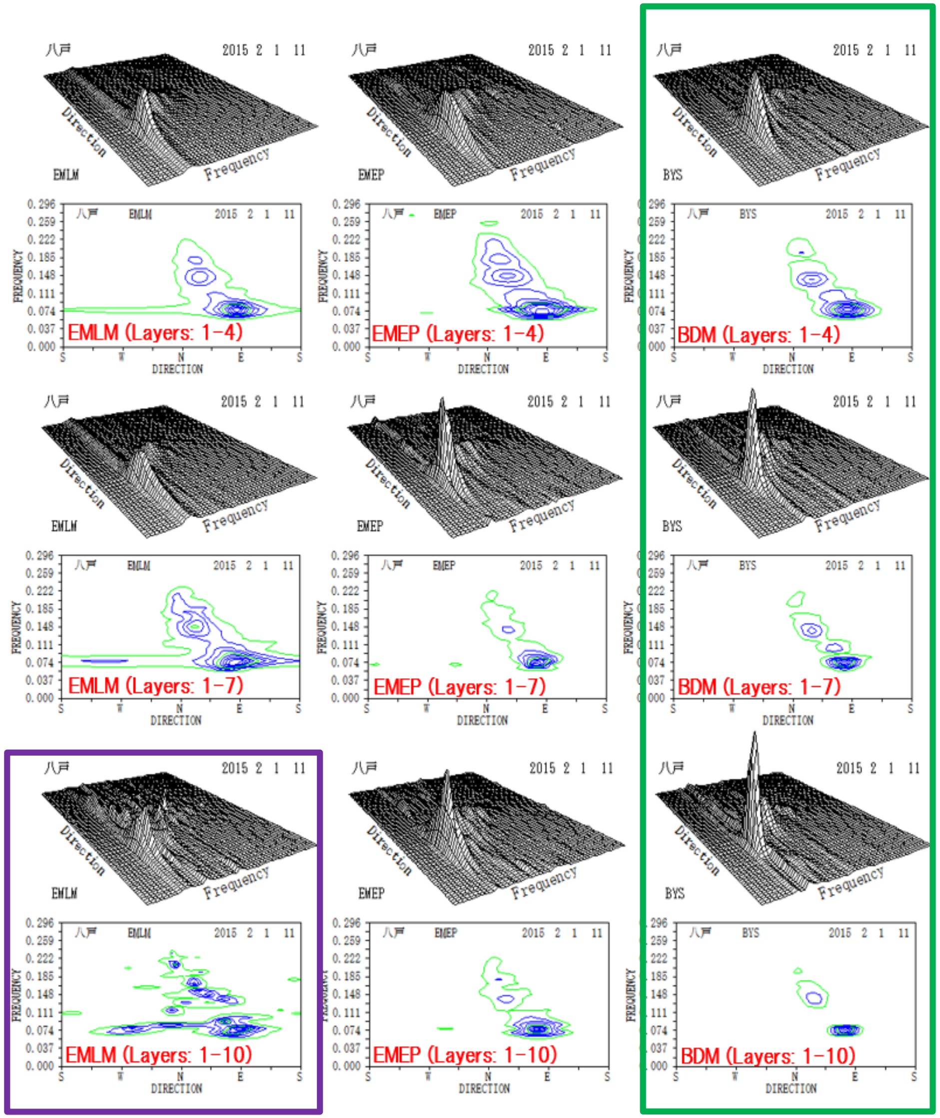
Examples of Directional Spectra at Hachinohe Port

Many components (a water surface elevation and many water particle velocities at several layers) are used for directional spectrum analyses.

13 components used (4 layers)

22 components used (7 layers)

31 components used (10 layers)



BDM shows higher directional concentration than the other methods.

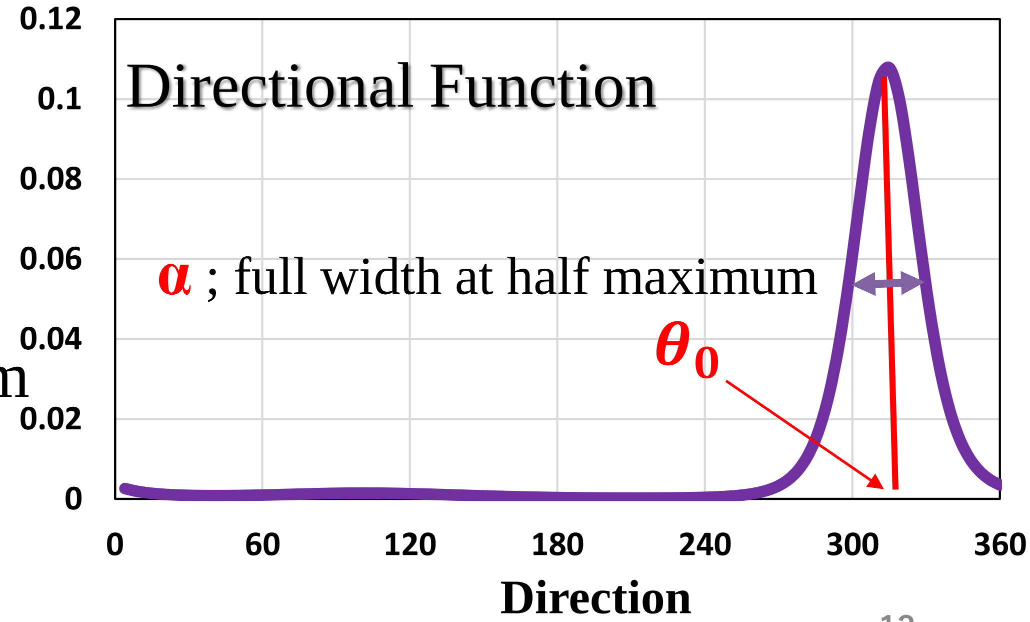
EMLM tends to estimate the directional energy distribution wider compared with the other methods.

In the case of using all of 10 layers, a multimodal unstable directional spectrum is estimated by EMLM.

Directional energy concentration Parameter $S_{\max}$

Parameter S indicates the directional energy concentration of waves

Direction energy concentration parameter $S_{\max}$ is calculated from the direction function at peak frequency f_p.



Directional function

$$G(f,\theta) = G_0 \cos^{2S}\left(\frac{\theta-\theta_0}{2}\right)$$

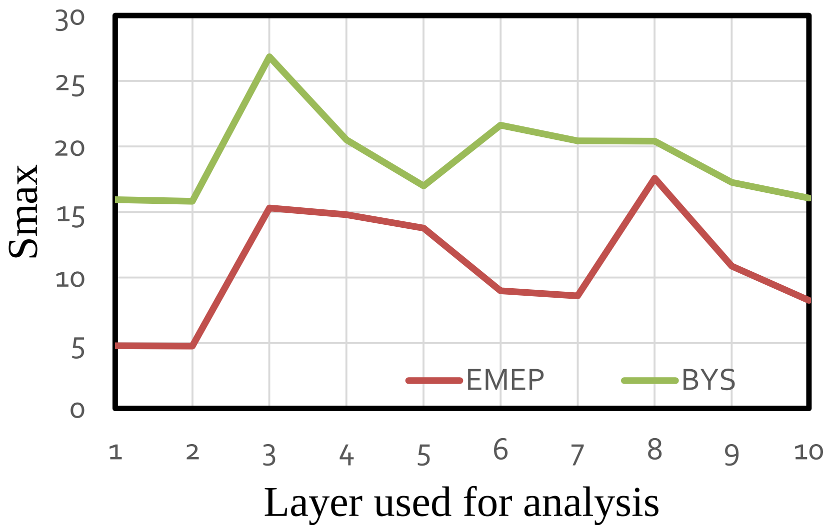
deformation of formula

Formula for calculating parameter **S**

$$S = \log\frac{1}{2} \Big/ \log\left(\frac{1+\cos(\alpha/2)}{2}\right)$$

α ; full width at half maximum

Comparison of $\boldsymbol{S}_{\mathbf{max}}$ estimated by BDM and EMEP for the cases using **four wave quantity components** of water surface elevation and three water particle velocity components measured at each layer from the uppermost layer (1st layer) to the lowermost layer (10th layer) respectively.

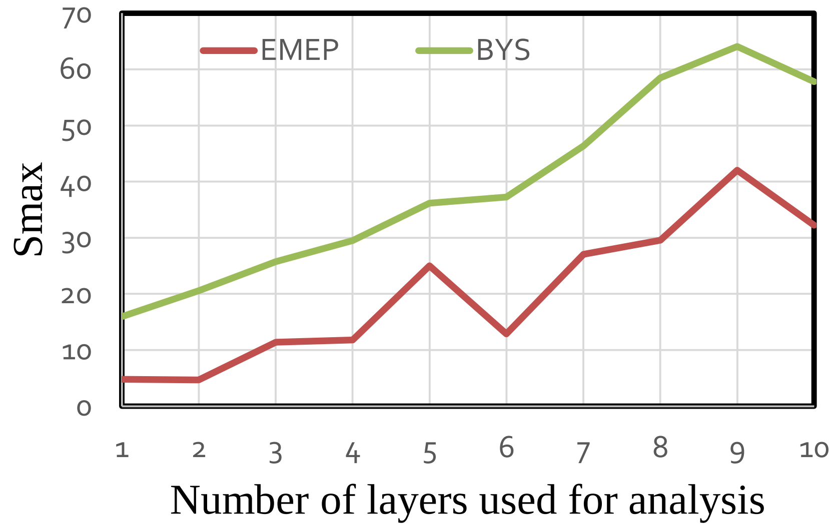


According to Goda, S_{max} of this data is estimated to be larger than 35 to 40 since $H_{1/3} = 2.02$ m, $T_{1/3} = 8.96$ s and $H_{1/3} / L_{1/3} = 0.018$.

$\boldsymbol{S}_{\mathbf{max}}$ greatly varies depending on the water depth of the water particle velocity components observed. $\boldsymbol{S}_{\mathbf{max}}$ estimated by BDM are higher than the those of EMEP, while the variation of the estimates are smaller in BDM.

(cf. Goda, Y., 2010, *Random Seas and Design of Maritime Structures*, World Scientific, 732pp.)

Comparison of S_{max} estimated by BDM and EMEP for the cases of increasing the number of layers of the water particle velocity components used for the directional spectrum estimations, in order from the upper 1st layer to the lower 10th layer.



S_{max} increases in proportion to the increase in the number of layers used.

Why ? Further research is necessary.

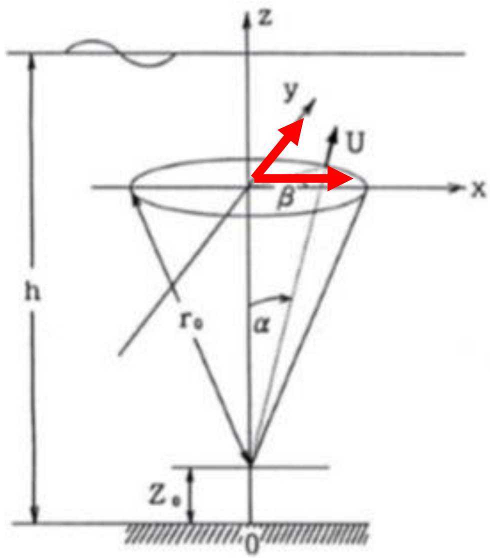
Transfer function derived from small amplitude wave theory

$$H(\alpha,\beta,r,h,z_0,\omega,\theta) = \frac{\omega \exp(-i\omega\Delta t)}{\sin kh}[\cosh\{k(r\cos\alpha + z_0)\} \times \sin\alpha\cos(\theta-\beta) - i\sinh\{k(r\cos\alpha + z_0)\} \times \cos\alpha]\exp\{ikr\sin\alpha\cos(\theta-\beta)\}$$

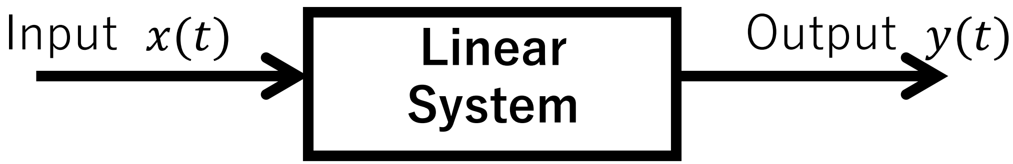
$$H(f,\theta) = [(H_u(f)\sin\alpha\cos(\theta-\beta) + iH_w(f)\cos\alpha] \times \exp[i(kr\sin\alpha\cos(\theta-\beta) - \omega\Delta t)]$$

$$H_u(f) = \omega\frac{\cosh\{k(r\cos\alpha + z_0)\}}{\sin kh} = \omega\frac{\cosh kz}{\sin kh}$$

$$H_w(f) = \omega\frac{\sinh\{k(r\cos\alpha + z_0)\}}{\sin kh} = \omega\frac{\sinh kz}{\sin kh}$$



Linear time series



$$y(t) = x(t) * h(\tau) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$$

$$H(f) = \int_{0}^{\infty} h(\tau)e^{-i2\pi f\tau}d\tau$$

$h(\tau)$:Response function

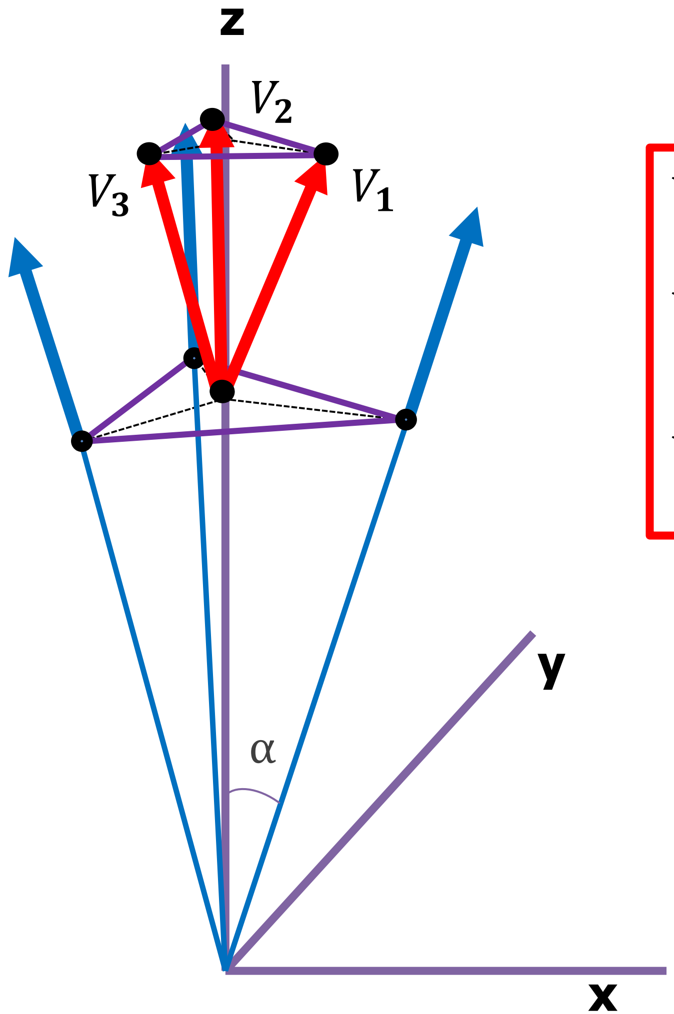
$H(f)$:Transfer function

$$Y(f) = \boldsymbol{H(f)}X(f)$$

$$\boxed{\boldsymbol{H(f) = \sqrt{\frac{S_y(f)}{S_x(f)}}}}$$

$S_x(f)$:The spectrum of the input $x(t)$

$S_y(f)$:The Spectrum of the output $y(t)$



$$V_1 = u \sin\alpha \cos\theta - iw \cos\alpha$$
$$V_2 = u \sin\alpha \cos(\theta - \frac{2}{3}\pi) - iw \cos\alpha$$
$$V_3 = u \sin\alpha \cos(\theta - \frac{4}{3}\pi) - iw \cos\alpha$$

$V_n (n = 1,2,3)$; Complex amplitude of each water particle velocity component

(u, w); Amplitude of horizontal and vertical components of water particle velocity

θ; Wave propagation direction, i ; imaginary unit

$$|V_1|^2 = u^2(\sin\alpha)^2(\cos\theta)^2 + w^2(\cos\alpha)^2$$
$$|V_2|^2 = u^2(\sin\alpha)^2\left(-\frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta\right)^2 + w^2(\cos\alpha)^2$$
$$|V_3|^2 = u^2(\sin\alpha)^2\left(-\frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta\right)^2 + w^2(\cos\alpha)^2$$

By eliminating θ from the above three equations, u and w are derived as

$$u = \sqrt{\frac{4\sqrt{|V_1|^4 + |V_2|^4 + |V_3|^4 - |V_1|^2|V_2|^2 - |V_2|^2|V_3|^2 - |V_3|^2|V_1|^2}}{3(\sin\alpha)^2}}$$
$$w = \sqrt{\frac{(|V_1|^2 + |V_2|^2 + |V_3|^2) \pm 2\sqrt{|V_1|^4 + |V_2|^4 + |V_3|^4 - |V_1|^2|V_2|^2 - |V_2|^2|V_3|^2 - |V_3|^2|V_1|^2}}{3(\cos\alpha)^2}}$$

Transfer function estimated from observed data

$$\boldsymbol{H_{New}(f,\theta)} = [(\boldsymbol{H_u(f)} \sin\alpha \cos(\boldsymbol{\theta}-\boldsymbol{\beta}) + \boldsymbol{i}\boldsymbol{H_w(f)} \cos\alpha] \times \exp[\boldsymbol{i}(\boldsymbol{kr} \sin\alpha \cos(\boldsymbol{\theta}-\boldsymbol{\beta}) - \boldsymbol{\omega}\Delta\boldsymbol{t})]$$

where,

$$\boldsymbol{H_u(f)} = \sqrt{\frac{4\sqrt{S_1^2(f)+S_2^2(f)+S_3^2(f)-S_1(f)S_2(f)-S_2(f)S_3(f)-S_3(f)S_1(f)}}{3(\sin\alpha)^2}}$$

$$\boldsymbol{H_w(f)} = \sqrt{\frac{\left(S_1(f)+S_2(f)+S_3(f)\right)-2\sqrt{S_1^2(f)+S_2^2(f)+S_3^2(f)-S_1(f)S_2(f)-S_2(f)S_3(f)-S_3(f)S_1(f)}}{3(\cos\alpha)^2}}$$

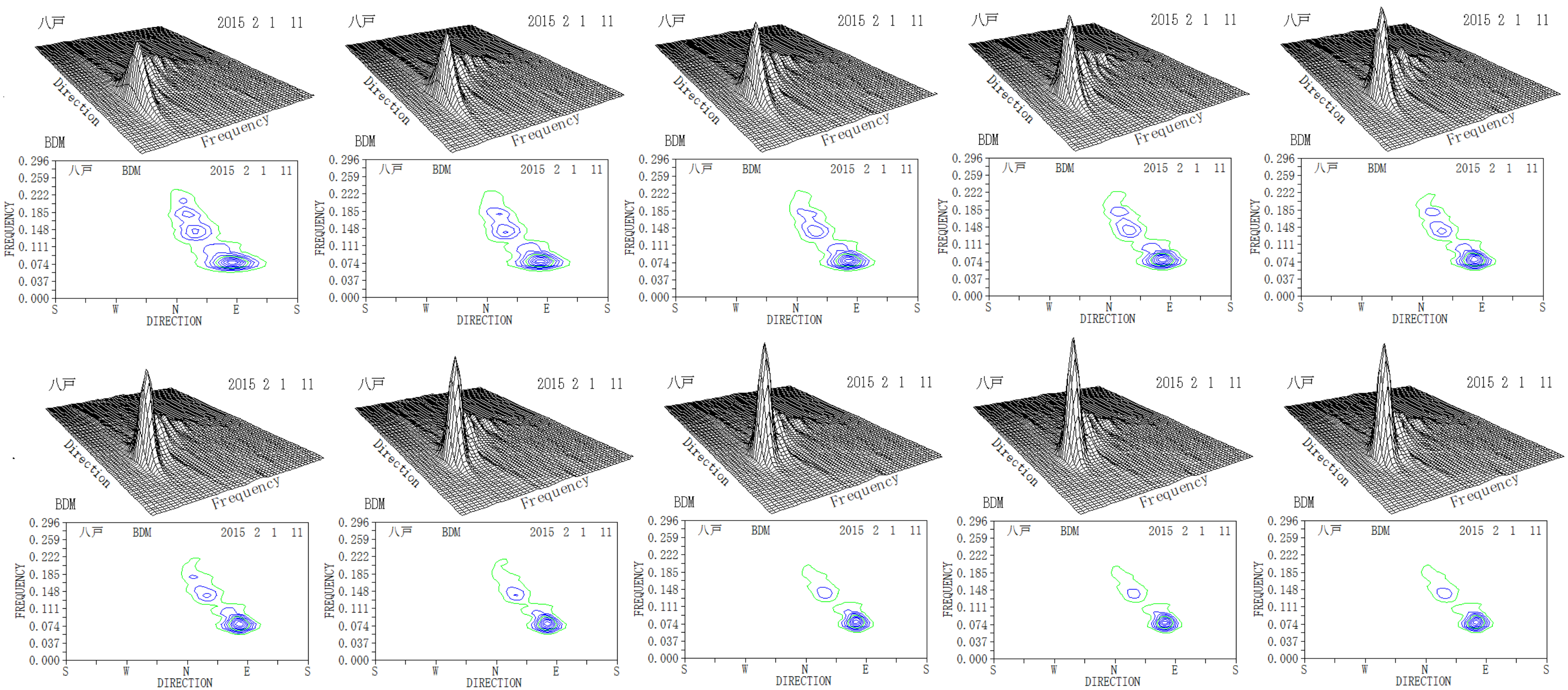
$$S_1(f) = S_{U_1}(f)/S_\eta(f)$$

$$S_2(f) = S_{U_2}(f)/S_\eta(f)$$

$$S_3(f) = S_{U_3}(f)/S_\eta(f)$$

$S_{U_i}(f)$; Spectrum of U_i

$S_\eta(f)$; Spectrum of η



Original transfer function

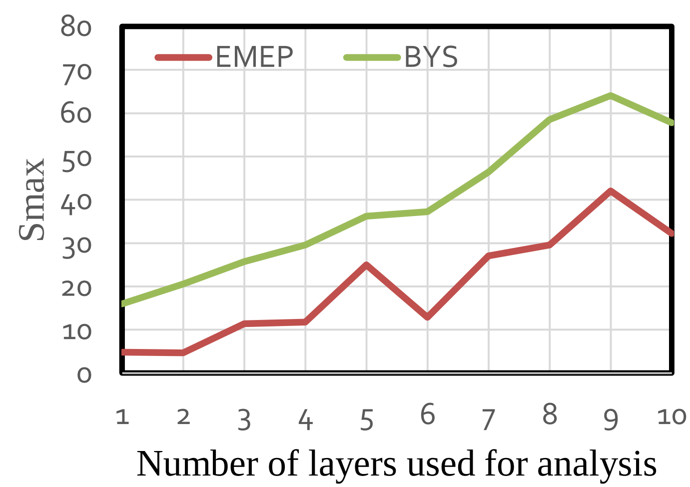
$$\overline{H}(\alpha,\beta,r_0,\Delta r,d,z_0;\omega,\theta)=\frac{-i\omega\exp(-i\omega\Delta t)}{\Delta r k\sinh kd}\Big[\cosh\{k(r\cos\alpha+z_0)\}$$

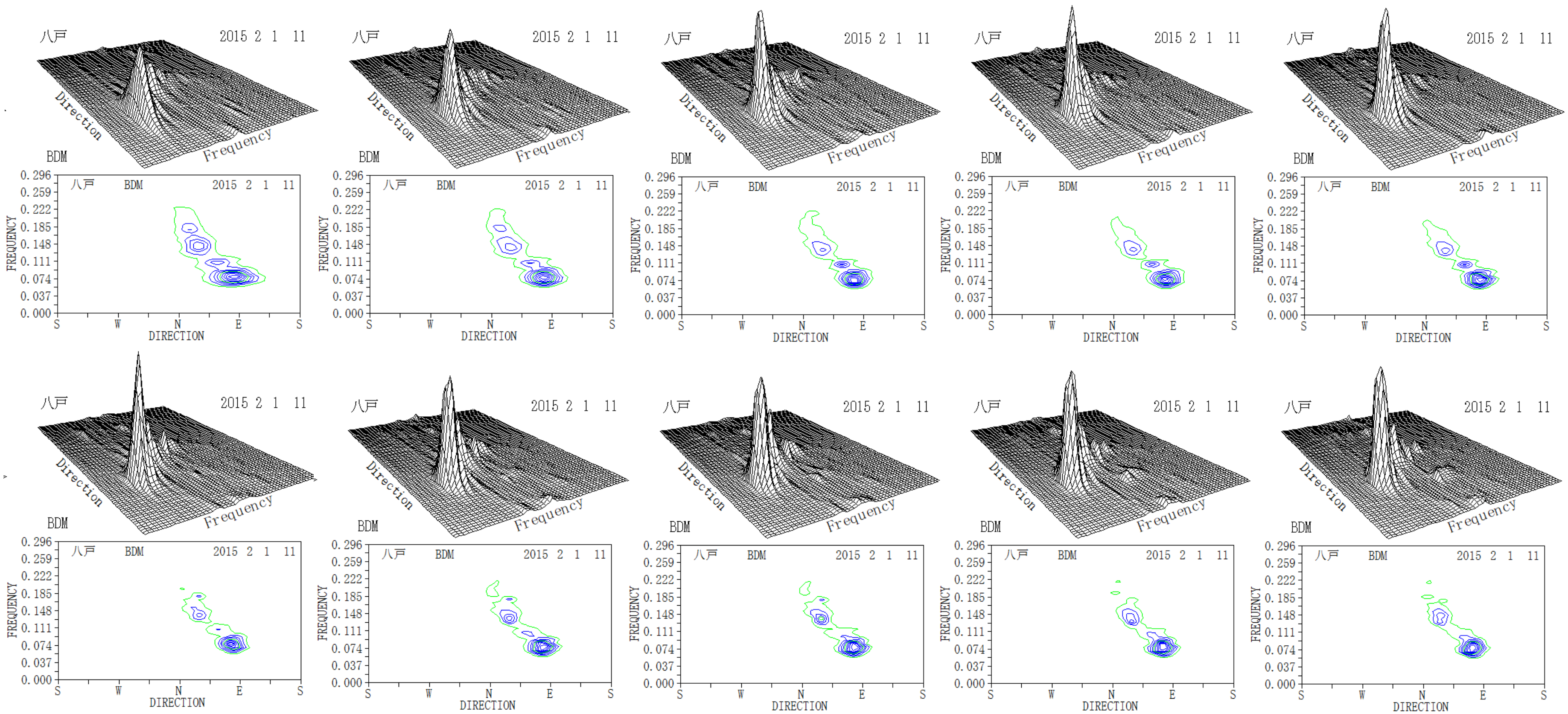
$$\times\exp\{ikr\sin\alpha\cos(\theta-\beta)\}\Big]_{r_0-\Delta r/2}^{r_0+\Delta r/2}.$$

Original weighting functions

$$W_{mn}=\sigma[\hat{C}_{mn}(f)]\approx\left[\left\{\Phi_{mm}(f)\Phi_{nn}(f)+C_{mn}(f)^2-Q_{mn}(f)^2\right\}/2N_a\right]^{1/2}$$

$$W_{mn}=\sigma[\hat{Q}_{mn}(f)]\approx\left[\left\{\Phi_{mm}(f)\Phi_{nn}(f)-C_{mn}(f)^2+Q_{mn}(f)^2\right\}/2N_a\right]^{1/2}$$





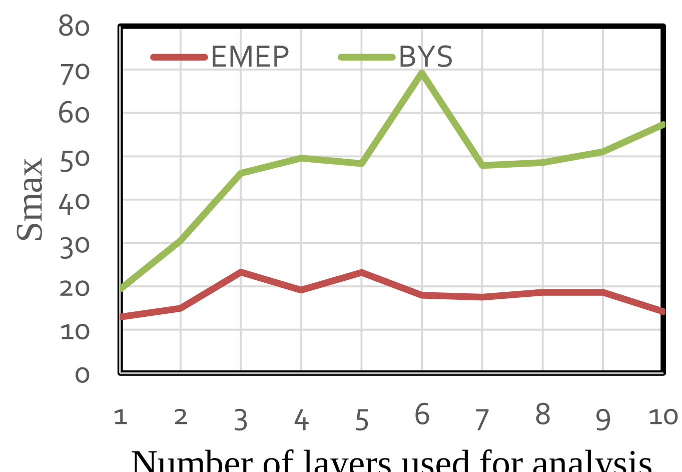
New transfer function

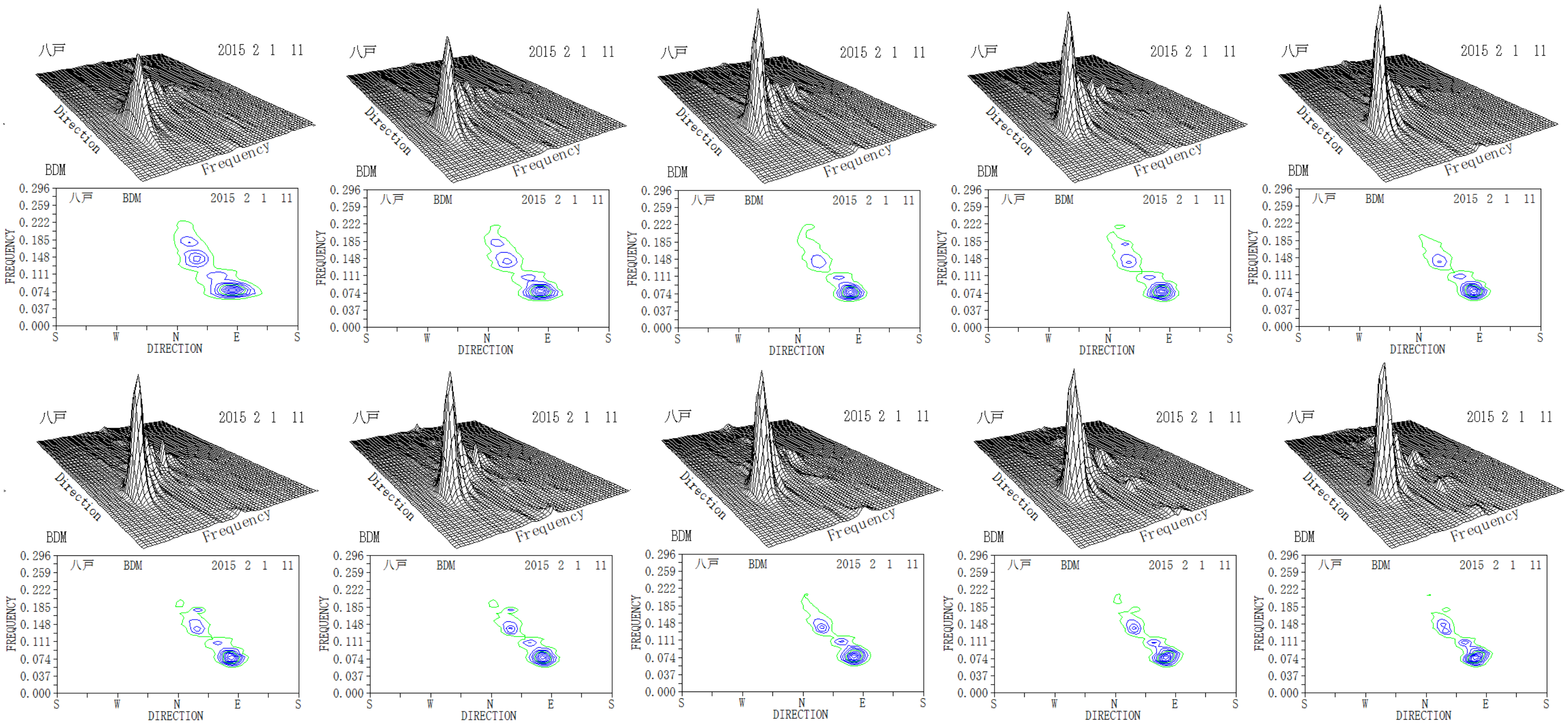
$$H_{New}(f,\theta) = [(H_u(f)\sin\alpha\cos(\theta-\beta) + iH_w(f)\cos\alpha] \times \exp[i(kr\sin\alpha\cos(\theta-\beta)-\omega\Delta t)]$$

Original weighting functions

$$W_{mn} = \sigma[\hat{C}_{mn}(f)] \approx \left[\left\{\Phi_{mm}(f)\Phi_{nn}(f) + C_{mn}(f)^2 - Q_{mn}(f)^2\right\}/2N_a\right]^{1/2}$$

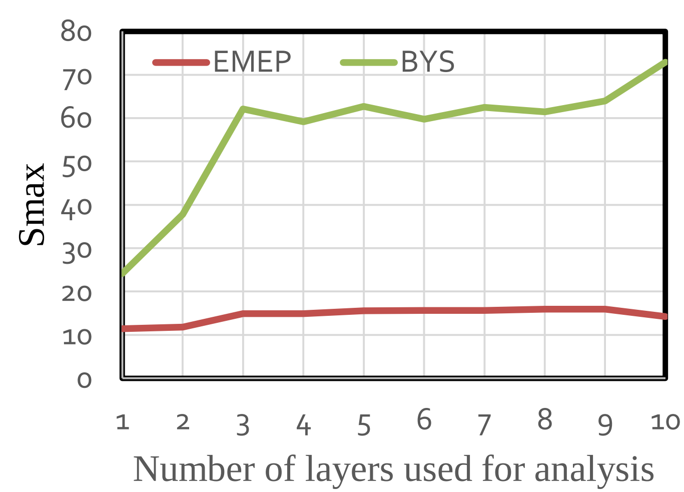
$$W_{mn} = \sigma[\hat{Q}_{mn}(f)] \approx \left[\left\{\Phi_{mm}(f)\Phi_{nn}(f) - C_{mn}(f)^2 + Q_{mn}(f)^2\right\}/2N_a\right]^{1/2}$$





New transfer function

$$H_{New}(f,\theta) = [(H_u(f)\sin\alpha\cos(\theta-\beta) + iH_w(f)\cos\alpha] \times \exp[i(kr\sin\alpha\cos(\theta-\beta) - \omega\Delta t)]$$



New weighting function

$$W_{mn} = [\Phi_{mm}(f)\Phi_{nn}(f)]^{1/2}$$

CONCLUSIONS

In order to estimate accurate directional spectra of ocean waves having infinite degrees of freedom with respect to frequency and direction, directional spectrum analysis was carried out using the data observed by DWM which is capable of measuring water surface elevation and 10 layers of water particle velocity components.

Initially, we had expected to be able to improve the estimation accuracy of the directional spectrum by increasing the number of wave quantities observed. Unfortunately, however, by increasing the number of observation layers with measuring water particle velocities at deeper water depth, it was found that the directional energy concentration tends to be higher.

In this study, we therefore investigated the transfer function and the weighting function in the equation for the directional spectrum estimation. As a result, we could improve accuracy and stability of the directional spectra observed with DWM.